change in the size of elementary areas over which the variations are correlated can have a significant effect. For example, if ΔA in this numerical example were 1 m^2 rather than 10 cm^2, the rms torque components would increase by a factor of 100, bringing them up to a more interesting level.

References

[1] Evans, W. J., "Aerodynamic and radiation disturbance torques on satellites having complex geometry," *Torques and Attitude Sensing in Earth Satellites*, edited by S. F. Singer (Academic Press Inc., New York, 1964), pp. 83–98.

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Boost-Phase Equilibrium Pressures in a Dual-Thrust Solid-Propellant Rocket Motor

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Nomenclature

a = (sustainer) burning-rate coefficient
A = area, in.^{2}
b = (booster) burning-rate coefficient
C_D = mass flow coefficient, lbm/lbf-sec
k = specific-heat ratio
m = (booster) pressure exponent
$\dot{m}$ = mass flow rate, lbm/sec
M = Mach number
n = (sustainer) pressure exponent
P = pressure, psia
r = burning rate, in./sec
R = gas constant, lbf-ft/lbm-°F
s = propellant surface area, in.^{2}
t = time, sec
T = temperature, °R
v = volume, in.^{3}
w = propellant web thickness, in.
ρ = density (of solid propellant), lbm/in.^{3}

Subscripts

b = booster
c = stagnation conditions
e = (nozzle) exit section
s = sustainer
si = subsonic (isentropic) sustainer nozzle
ss = sonic sustainer nozzle
$(\ )^*$ = (nozzle) throat section

AFTER boost to a desired velocity, a sustain capability equal to drag allows a vehicle to fly a "vacuum" trajectory, thereby increasing both range and accuracy. Providing a sustain capability requires some form of dual-thrust propulsion system; the use of a dual-nozzle configuration (Fig. 1) offers one solution. Simultaneous boost-sustain ignition allows maximum reliability by eliminating a dual-ignition system. However, qualitative consideration of the combustion process indicates that the boost chamber pressure could impede the sustainer flow, causing some sustainer overpressurization. In turn, the sustainer gases entering the boost chamber cause the boost pressure to rise above design values. This note examines the equilibrium pressures attained in each motor during the boost phase and defines the influencing parameters.

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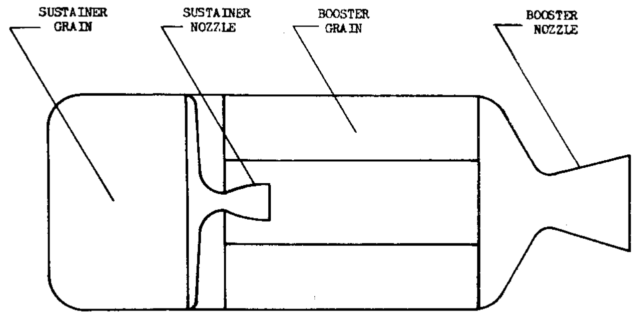

Fig. 1 Dual-nozzle rocket motor.

Design Parameters

Sustain thrust is 4% of boost, with both motors designed for 1000 psia P_c. Thus sustainer operation is influenced largely by the flow characteristics of a converging-diverging nozzle discharging into a region of variable back pressure.[1] If $P_{cs} \lesssim 1.1\ P_{cb}$, the nozzle acts as a subsonic venturi. For higher P_{cs}, $M = 1.0$ at the nozzle throat, the flow rate is constant (independent of P_{cb}), and P_{cb} is achieved through a normal shock in the divergent section.

Transient State

Assuming one-dimensional isentropic flow of a perfect gas through the combustion chamber, the conservation of mass yields the usual[2] solid-propellant design equation:

$$\frac{P_c}{RT} rS + \frac{v}{RT^2}\left(T\frac{dP_c}{dt} - P_c\frac{dT}{dt}\right) = \dot{m}_{\text{in}} - \dot{m}_{\text{out}} = \rho rS - C_D P_c A_* \quad (1)$$

Assuming negligible variations in combustion temperature, and burning rate given by $r = aP_c{}^n$, Eq. (1) becomes

$$\frac{dP_c}{dt} = \frac{RT}{V}\left[\rho a P_c{}^n S\left(1 - \frac{P_c}{\rho RT}\right) - C_D P_c A_*\right] \quad (2)$$

where $V = V_0 + \int aP_c{}^n S dt$.

From (2), the rate of change of boost and sustain chamber pressures are given by

$$\frac{dP_b}{dt} = \frac{R_bT_b}{V_b}\left[bP_b{}^m S_b\left(\rho_b - \frac{P_b}{R_bT_b}\right) + \dot{m}_s - C_{Db}P_bA_{b*}\right] \quad (3)$$

$$\frac{dP_s}{dt} = \frac{R_sT_s}{V_s}\left[aP_sS_s\left(\rho_s - \frac{P_s}{R_sT_s}\right) - \dot{m}_s\right] \quad (4)$$

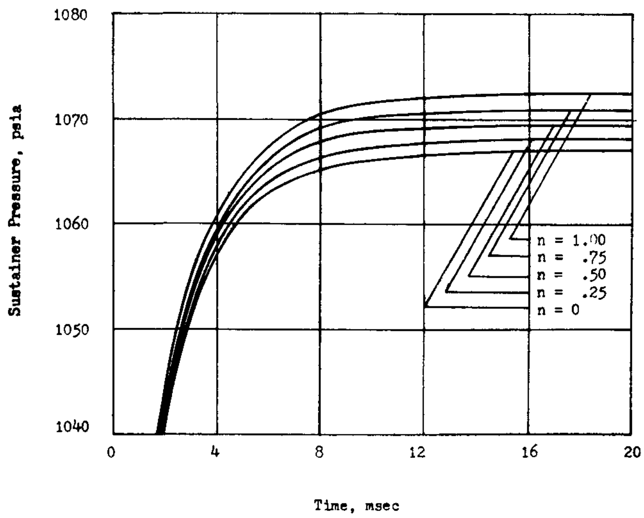

Fig. 2 Transient-state sustainer pressures for various values of pressure exponent (n).

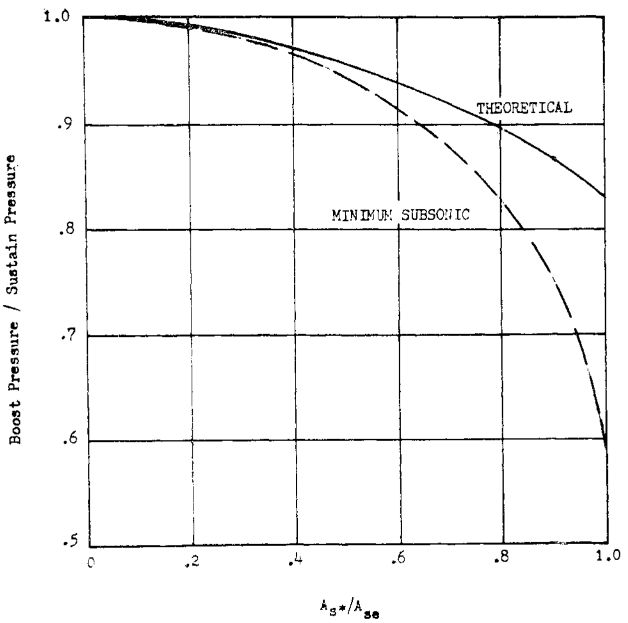

Fig. 3 Theoretical P_b/P_s and minimum pressure ratio for subsonic flow in the sustainer nozzle as functions of sustainer nozzle area ratio.

where $\dot{m}_s$ is the mass flow through the sustainer nozzle, which can be either subsonic or sonic at the throat (depending upon whether P_b/P_s is greater or less than the "critical" subsonic value determined by the usual function of nozzle area ratio). If $M = 1.0$ at the sustainer throat,

$$\dot{m}_s = C_{Ds} P_s A_{s*} \tag{5}$$

If $M < 1.0$ at the throat,

$$\dot{m}_s = \frac{P_b A_{se}}{R_s T_s \left(\frac{P_b}{P_s}\right)^{k-1/k}} \left\{\frac{2gkR_sT_s}{k-1}\left[1-\left(\frac{P_b}{P_s}\right)^{k-1/k}\right]\right\}^{0.5} \tag{6}$$

Upon determining the proper $\dot{m}_s$, (3) and (4) are solved simultaneously for boost and sustain pressure as functions of time.

Assuming both chambers initially at 1000 psia, Fig. 2 gives sustainer pressure as a function of time to equilibrium for values of $n = 0$, 0.25, 0.50, 0.75, and 1.0. In each instance P_b/P_s is approximately 0.9972, slightly larger than the "critical" subsonic pressure ratio of 0.997 determined from the nozzle area ratio (A_{se}/A_{s*}) of 8.292 (Fig. 3). Thus sustainer nozzle flow is seen to be subsonic during booster burning.

Equilibrium State

Setting $dP/dt = 0$ in (3) and (4) yields expressions for the boost and sustain equilibrium pressures:

$$\dot{m}_s = C_{Db} P_b A_{b*} - bP_b{}^m S_b(\rho_b R_b T_b - P_b)/R_b T_b \tag{7}$$

$$\dot{m}_s = aP_s{}^n S_s(\rho_s R_s T_s - P_s)/R_s T_s \tag{8}$$

Table 1 Equilibrium pressures, $P_{bo} = P_{so} = 1000$ psia

Sustainer pressure exponent n	Nozzle area ratio, A_{sc}/A_{s*}	Booster equilibrium pressure, psia	Sustainer equilibrium pressure, psia
0	8.29	1064	1067
...	1.0	1064	1271
0.25	8.29	1065	1068
...	1.0	1069	1306
0.50	8.29	1067	1070
...	1.0	1078	1357

Table 2 Experimental results

	Case A $P_{bo} = 1000$, $P_{so} = 1200$		Case B $P_{bo} = 1200$, $P_{so} = 1200$	
rt/w	P_b, psia	P_s, psia	P_v, psia	P_s, psia
0.2	1074	1195	1295	1300
0.4	1067	1205	1315	1320
0.6	1061	1205	1320	1310
0.8	1048	1205	1295	1305
1.0	1022	1210	1250	1260

Equating (7) and (8) gives an implicit equation for P_s as a function of P_b which is solved using a general trial-and-error convergence routine. Obviously, different values of P_b give different values of P_s. The procedure used to determine the actual equilibrium value is given below.

With a "required" P_s from the foregoing, the pressure P_{ss} at which the sustainer would operate for $M = 1.0$ at the throat is $P_{ss} = \dot{m}_s/C_{Ds}A_{s*}$, and the operating pressure (P_{si}) for subsonic (isentropic) is given from

$$P_{si} = P_b/(x)^{k/k-1} \tag{9}$$

where

$$x = [(1 + 4f)^{0.5} - 1]/2f$$

$$f = (\dot{m}_s/d)^2$$

$$d = P_b A_{se}[2gk/(k - 1)R_s T_s]^{0.5}$$

This procedure is repeated, increasing P_b by an arbitrary increment (dP_b), until a value of P_s less than P_b is calculated. The two possible sustainer operating pressures P_{ss} and P_{si} are then inspected at (P_b) and ($P_b - dP_b$) to determine the actual equilibrium pressure.

Table 1 gives the equilibrium pressures obtained with the foregoing method for a booster (and sustainer) designed for 1000 psia. Comparison with the transient results (Fig. 2) shows excellent agreement. As indicated, the influence of n is small compared to that of A_{se}/A_{s*}.

Experimental Results

Analytical predictions were verified by experimental data obtained from system performance tests. Table 2 (case A) gives results typical of those obtained with a 1000 psia booster and a sustainer designed for operation at 1200 psia with a nozzle area ratio of 8.3. With the design $P_b/P_s = 0.833$, sonic flow would be expected at the nozzle throat with a downstream shock. As indicated, P_s appears unaffected by booster operation, within the accuracy of the data. Case B gives results typical of those obtained with a booster redesigned for 1200 psia operation. As predicted, P_b/P_s is compatible with the subsonic flow condition, and the influence of the boost back pressure is evident.

Summary

Two methods of calculation are used to predict maximum operating pressures in a dual-nozzle propulsion system wherein the sustain motor exhausts into the boost motor combustion chamber. Transient-state calculations for equilibrium pressures are compared to direct calculations for the equilibrium state, with excellent agreement.

Generally, it is found that booster equilibrium pressure is a definite function of sustainer mass flow rate; sustainer equilibrium pressure is independent of nozzle geometry (A_{se}/A_{s*}) and boost pressure if $M = 1.0$ at the sustainer throat; if $M < 1.0$ at the sustainer throat, sustainer pressure is a significant function of A_{se}/A_{s*}, boost pressure, and sustainer throat area. A limited amount of experimental data has verified the theoretical predictions.

References

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Supersonic Turbines in Space Power Applications

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Nomenclature

C_f = skin-friction coefficient
c_p = specific heat at constant pressure, kcal/(kg-°K)
g = acceleration of gravity, m/s^2
J = mechanical equivalent of heat = 427 mkg/kcal
k = thermal conductivity, cal/(cm-s-°K)
$\mathfrak{M}$ = gas-molecular weight
p = pressure, kg/cm^2
Pr = Prandtl number, $c_p\mu/k$
q = heat-transfer rate per unit area, kcal/(m^2 − s)
$\mathfrak{R}$ = universal gas constant, kcal/(kmole-°K)
r = recovery factor
r_c = compressor pressure ratio
St = Stanton number
T = absolute temperature, °K
u = blade peripheral speed, m/s
V = absolute velocity, m/s
W = relative velocity, m/s
α = angle between u and V (less than 90°)
γ = isentropic exponent
μ = viscosity coefficient, poise
ρ = density, kg/m^3

Subscripts

0 = stagnation
1 = absolute
2 = relative
3 = compressor inlet
aw = adiabatic wall
e = boundary-layer edge
w = wall

PEAK temperatures in both Rankine and Brayton cycles for energy conversion in space are limited by the heat-source characteristics, as well as by the peak temperature at which highly loaded components can operate safely. The use of supersonic turbines represents a means to lower the actual operating temperature of the turbine, which is the most critical component of high-temperature turbo-converters.

Table 1 Some properties of working fluids for space power applications

Gas or vapor	γ	Pr
Argon	1.668	0.662[a]
Krypton	1.680	0.683[b]
Xenon	1.660	0.683[b]
Xe-He Mixture[c]	1.667	0.203[d]
Methane	1.310	0.210[a]
Ethane	1.220	0.240[a]
Na, K, Cs, Hg	1.667	0.683[b]

[a] At 1500°K.
[b] Estimated[6] from $Pr = 4\gamma/(9\gamma - 5)$.
[c] 50.6% Xenon-49.4% Helium mixture; $\mathfrak{M}$ = 68.5.
[d] At 288°K.

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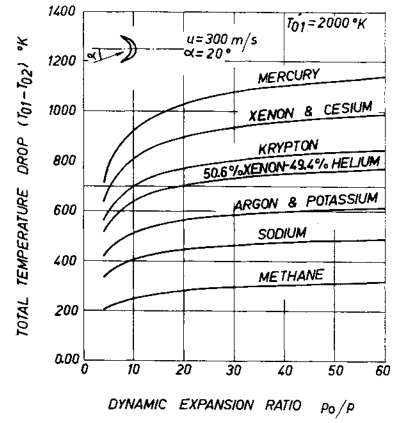

Fig. 1 Total temperature drop passing from stationary nozzles to turbine-rotating channels.

The basic characteristics of supersonic flow in turbine distributors and rotating stages are as follows: Assume that a perfect gas with constant specific heats expands isentropically from stagnation conditions characterized by a temperature T_{01} and a pressure p_0 to a pressure p. The exit velocity of the flow is given by

$$V = \left\{\frac{2\gamma g J \mathfrak{R}}{(\gamma - 1)\mathfrak{M}} T_{01}\left[1 - \left(\frac{p}{p_0}\right)^{\gamma - 1/\gamma}\right]\right\}^{1/2} \quad (1)$$

The energy equation allows the calculation of the static temperature T

$$T = T_{01} - V^2/2gJc_p \quad (2)$$

where c_p is the specific heat per unit weight, another expression of which is

$$c_p = \gamma\mathfrak{R}/\mathfrak{M}(\gamma - 1) \quad (3)$$

In a turbine-rotating channel, which the flow enters at a relative velocity W, the energy equation is

$$T_{02} = T + W^2/2gJc_p \quad (4)$$

where T_{02} is the stagnation temperature in a rotating frame of reference.

The relative velocity is given by

$$W^2 = V^2 + u^2 - 2uV\cos\alpha \quad (5)$$

in which u is the turbine peripheral speed, and α is the angle, less 90°, between u and V.

Combining Eqs. (2–5), one obtains the stagnation-temperature drop, passing from stationary to rotating channels

$$T_{01} - T_{02} = \frac{2uV\cos\alpha - u^2}{2\gamma g J\mathfrak{R}/\mathfrak{M}(\gamma - 1)} \quad (6)$$

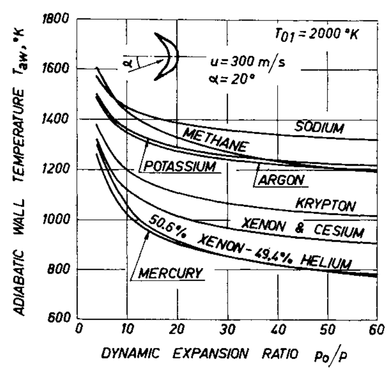

Fig. 2 Recovery temperature of adiabatic rotating blades vs stationary nozzles pressure ratio.